Oral Qualifying Exam Syllabus

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1 Analytic Number Theory

- Poisson summation, Mellin transform.
- Dirichlet characters, primitivity, orthogonality relations, Gauss sums.
- The θ function and θ_χ, the Riemann ζ function and Dirichlet L-functions, Euler products, analytic continuation, location of zeros/poles, functional equations.
- Analytic proof of the infinitude of primes and Dirichlet's theorem on primes in arithmetic progressions. Prime number theorem w/ basic consequences and prime number theorem for arithmetic progressions. Statement of Bomberi-Vinogradov.
- Λ^2-Sieve w/ application to Brun's theorem, the large sieve, short character sums.

[Dav80] Chapters 4, 8, 9, 13, 14, 18, 20 (Non-sieve content)

[IK04] Chapters 6 & 7 (Sieve content)

2 Modular Forms

- Modular/cusp forms, notable examples (Eisenstein Series, Δ-function, Poincare Series), Fourier expansions.
- The space of M_k and S_k, arithmetic identities on σ_k arising from $\dim_{\mathbb{C}} M_k = 1$, computing $\tau(n)$.
- The Hecke bound, L-functions of cusp forms in S_k, functional equations.
- Hecke operators, Petersson inner product, self-adjointness of Hecke operators w.r.t Petersson inner product.
- Atkin-Lehner theory i.e. spaces of oldforms and newforms, L-functions of Hecke eigenforms in $S_k(\Gamma_0(N))$, functional equations, Euler product of L-functions associated to newforms.

[Iwa97] Chapters 1, 3, & 6

References

[Dav80] Harold Davenport. *Multiplicative number theory.* 1980.

[IK04] Henryk Iwaniec and Emmanuel Kowalski. *Analytic number theory.* 2004.

[Iwa97] Henryk Iwaniec. *Topics in classical automorphic forms.* 1997.