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**Math Club: Biweekly Contest Week Five**

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**Release Date:** November 8, 2023

**Instructions:** Solve the following problem as best you can. The first student to submit the correct solution via email to tamumathcontest@gmail.com or to Jeremy Kubiak in Blocker 336D (with time stamp) wins!

**Problem 1.** Let  $M_2$  be the group of invertable  $2 \times 2$  matrices. Recall that we have both matrix addition and matrix multiplication on  $M_2$ , these operations are *commutative* and *non-commutative* respectively. That is, for  $a, b \in M_2$  we have that  $a + b = b + a$  and  $ab \neq ba$ . Finally, recall that a matrix times its inverse is the identity matrix. That is for  $a \in M_2$  we have that  $aa^{-1} = a^{-1}a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ; additionally, if a matrix is a multiple of the identity matrix we call it a *constant*.

Let  $x, y \in M_2$ , compute the constant term of the expression  $(x + x^{-1} + y + y^{-1})^4$ .