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### Math Club: Biweekly Contest Week Three

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**Release Date:** September 27, 2023

**Instructions:** Solve the following problem as best you can. The first student to submit the correct solution via email to tamumathcontest@gmail.com or to Jeremy Kubiak in Blocker 336D (with time stamp) wins!

**Problem 1.** Let  $a, b, c, d, e$  be a permutation of the set  $\{1, 2, 3, 4, 5\}$  (without repetition).

Find the number of permutations such that  $a < b > c < d > e$  or  $a > b < c > d < e$ .

**Solution.** Lets first work with the case that  $a < b > c < d > e$ . Note that either  $b$  or  $d$  must be 5. WLOG, assume that  $b = 5$ . Then  $d$  is either 3 or 4. If  $d = 3$ , then  $e = 4$  necessarily and there are a remaining  $2!$  to choose  $a$  and  $c$ . Otherwise, if  $d = 4$ , then there is a remaining  $3!$  ways to choose  $a, b$ , and  $c$ . So if we choose  $b = 5$  then there is  $8 = 2 + 6 = 2! + 3!$  ways to choose  $a, c, d$ , and  $e$ . By symmetry (between choosing  $b$  or  $d$  to be 5) we have that there are 16 permutations such that  $a < b > c < d > e$ . By symmetry there are 32 permutations such that  $a < b > c < d > e$  or  $a > b < c > d < e$ . Note that there is no overlap in this counting; suppose there was, then both  $a < b$  and  $b > a$  which is a contradiction.