

Name: _____

Math Club: Contest Week Five

Release Date: April 5, 2023

Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. Suppose we have 11 identical wooden blocks $2\text{ cm} \times 3\text{ cm} \times 5\text{ cm}$ in dimension and wish to build a tower 32 cm high. If the blocks are stacked flat on top of each other with random orientations what is the probability that after all 11 blocks are stacked a 32 cm tower is produced.

Solution. The conditions of the problem give us the system of Diophantine equations

$$2a + 3b + 5c = 32$$

$$a + b + c = 11.$$

By eliminating the variable b , we know that $b + 3c = 10$. Thus, $(b, c) = (10, 0), (7, 1), (4, 2)$, or $(1, 3)$. By plugging in (b, c) into $a + b + c = 11$ we get $(a, b, c) = (1, 10, 0), (3, 7, 1), (5, 4, 2)$, or $(7, 1, 3)$. Also note that,

$$2(1) + 3(10) + 5(0) = 32$$

$$2(3) + 3(7) + 5(1) = 32$$

$$2(5) + 3(4) + 5(2) = 32$$

$$2(7) + 3(1) + 5(3) = 32.$$

So we have computed the complete set of solutions to this Diophantine equation. Note that there are $11!/(a!b!c!)$ ways to create a stack with a, b , and c blocks contributing 2 cm, 3 cm, and 5 cm to the total height respectively. So,

$$(1, 10, 0) \rightarrow \frac{11!}{1!10!0!} = 11 \text{ ways}, \quad (3, 7, 1) \rightarrow \frac{11!}{3!7!1!} = 1320 \text{ ways}$$

$$(5, 4, 2) \rightarrow \frac{11!}{5!4!2!} = 6930 \text{ ways}, \quad (7, 1, 3) \rightarrow \frac{11!}{7!1!3!} = 1320 \text{ ways}.$$

Noting that there are 3^{11} total ways to stack the blocks, we get our final probability

$$\frac{11 + 1320 + 6930 + 1320}{3^{11}} = \frac{9581}{177147} \approx 5.409\%$$