Math Club: Contest Week Four

Release Date: March 22, 2023

Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. Use the identity for the determinant of a 4×4 Vandermonde matrix,

$$\det \begin{pmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{pmatrix} = (x_4 - x_3)(x_4 - x_2)(x_4 - x_1)(x_3 - x_2)(x_3 - x_1)(x_2 - x_1)$$

to compute

$$\det \begin{pmatrix} 1 & 7 + 2i & 45 + 28i & 259 + 286i \\ 1 & 5 - 3i & 16 - 30i & -10 + 198i \\ 1 & 3 + 5i & -16 + 30i & -198 + 10i \\ 1 & 2 - 7i & -45 - 28i & -286 + 259i \end{pmatrix}.$$

Solution. Simply doing the necessary computation we get,

$$= (x_4 - x_3)(x_4 - x_2)(x_4 - x_1)(x_3 - x_2)(x_3 - x_1)(x_2 - x_1)$$
$$= (2 - 7i - 3 - 5i)(2 - 7i - 5 + 3i)(2 - 7i - 7 - 2i)(3 + 5i - 5 + 3i)(3 + 5i - 7 - 2i)(5 - 3i - 7 - 2i)$$
$$= (-1 - 12i)(-3 - 4i)(-5 - 9i)(-2 + 8i)(-4 + 3i)(-2 - 5i)$$
$$= (-45 + 40i)(82 - 22i)(23 + 14i) = (-2810 + 4270i)(23 + 14i) = -124410 + 58870i.$$