

Name: _____

Math Club: Contest Week Two

Release Date: February 1, 2023

Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. We define the p -adic valuation $v_p : \mathbb{Z}_{>0} \rightarrow \mathbb{Z}_{\geq 0}$

$$v_p(n) = \max(k \in \mathbb{Z}_{\geq 0} : p^k \text{ divides } n)$$

for all primes p . Make an argument to show that for all $n \in \mathbb{Z}_{>0}$,

$$v_p(1) + v_p(2) + \dots + v_p(p^n) = \frac{1 - p^n}{1 - p}.$$

Hint. Here are some example p -adic valuations for reference.

- $v_5(375) = 3$ because 5^3 is the greatest power of 5 which divides into 375.
- $v_7(833) = 2$ because 7^2 is the greatest power of 7 which divides into 833.
- $v_{11}(4719) = 2$ because 11^2 is the greatest power of 11 which divides into 4719.