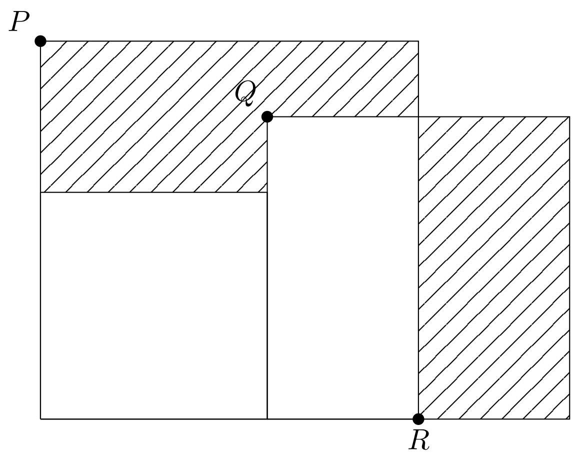
Name: ____________________

Math Club: Contest Week Seven

Release Date: November 30, 2022

Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. Three squares with side lengths $a < b < c$ are arranged as below



If the hatched areas are equal, what is $\angle PQR$?

Solution. Let us first compute how a, b, and c relate. The area of the upper-most hatched area is

$$c(c-b)+(b-a)a = c^2 - cb + ba - a^2.$$

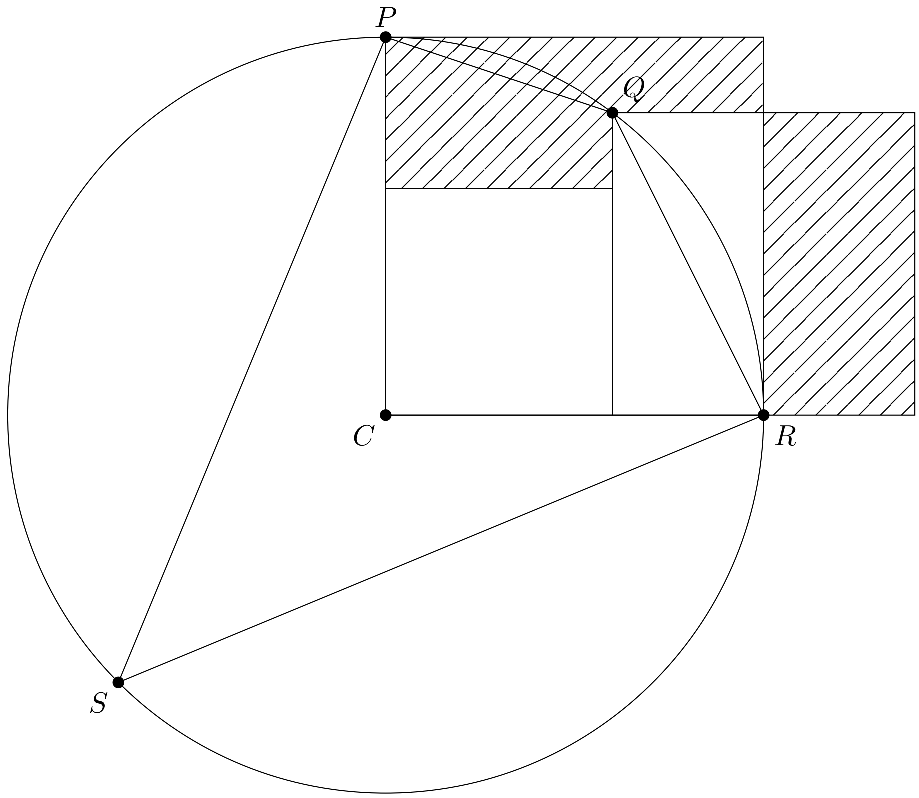
The area of the right-most hatched area is

$$b(b-c+a) = b^2 - cb + ba.$$

Thus

$$c^2 - cb + ba - a^2 = b^2 - cb + ba \implies c^2 = a^2 + b^2.$$

This implies that P, Q, and R all lie on a circle with radius c centered at the bottom left corner as shown. We also add some points and lines and explain their construction (see next page).



We add a point S on the circumference of the circle between P and R but opposite Q (its position on this arc does not matter), and we also construct the cyclic quadrilateral $PQRS$. We recall a basic geometric theorem: when two angles are subtended by the same arc, the angle at the center of the circle is twice that the angle on the circumference. Thus, $\angle PCR = 2\angle PSR$; since $\angle PCR = 90^\circ$, we know that $\angle PSR = 45^\circ$. We recall another basic geometric theorem: opposite angles in a cyclic quadrilateral must add to 180°. Thus, $\angle PSR + \angle PQR = 180^\circ$; since $\angle PSR = 45^\circ$, we know that $\angle PQR = 135^\circ$.