Name: _____

Math Club: Contest Week Six

Release Date: November 16, 2022
Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. Show that 53 divides every number of the form

$$\begin{aligned} &1007\\ &10017\\ &100117\\ &1001117\\ &10011117\\ &\vdots \end{aligned}$$

Solution. Let us assign varibles to this sequence

$$\begin{aligned} a_1 &= 1007\\ a_2 &= 10017\\ a_3 &= 100117\\ a_4 &= 1001117\\ a_5 &= 10011117\\ &\vdots \end{aligned}$$

We prove the desired result by induction. For the base case $k = 1$; note,

$$19 \cdot 53 = 1007.$$

Thus, 53 divides a_1. For the induction step, note that for all natural k, $a_{k+1} = 10a_k - 53$. Thus, if 53 divides a_k, then 53 also divides $10a_k - 53$ and a_{k+1}. This is sufficient to show the desired result.