

Name: _____

Math Club: Contest Week Two

Release Date: September 21, 2022

Instructions: Solve the following problem the best you can, first to submit the correct solution via email or the secretaries in Room 332 (with time stamp) wins!

Problem 1. We define a binary operator $*$ on the integers with properties

$$x * x = 0 \quad \text{and} \quad (x * y) + z = x * (y * z)$$

for all x , y , and z . Compute $2202 * 2022$.

Solution. Note that for all x

$$x * 0 = x * (x * x) = (x * x) + x = 0 + x = x.$$

Note that for all x and y

$$(x * y) + y = x * (y * y) = x * 0 = x.$$

Thus, $x * y = x - y$. Then $2202 * 2022 = 2202 - 2022 = 180$.